

# Pareto Front Exploration for Parametric Temporal Logic Specifications of Cyber-Physical Systems

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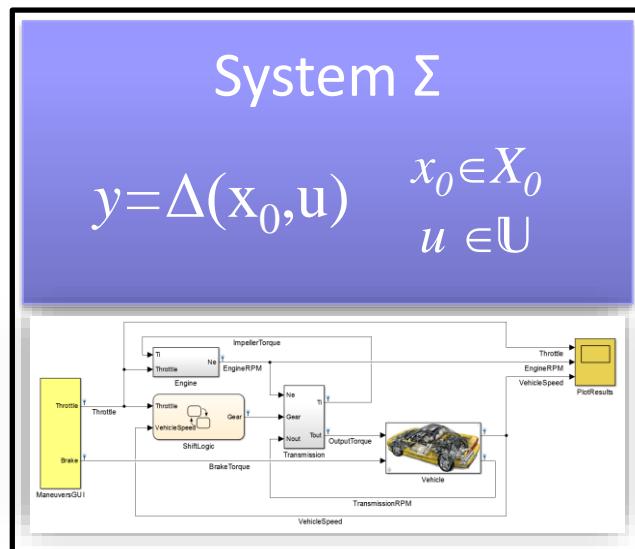
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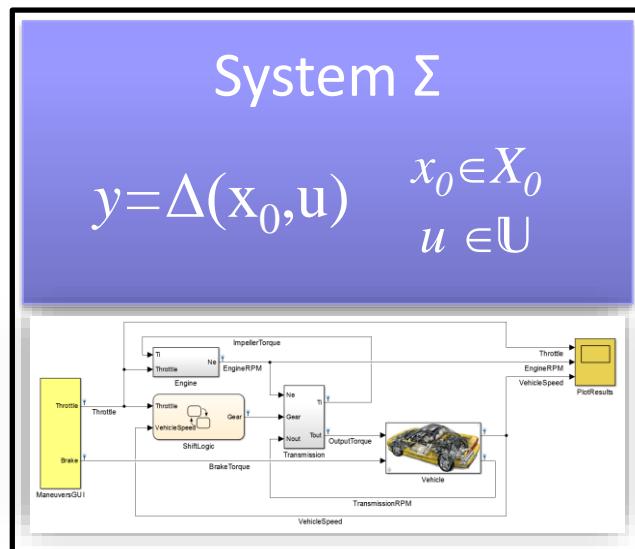
 <http://www.public.asu.edu/~bhoxha>

# Parameter Mining



# Parameter Mining

What is the shortest time  
that the engine speed can  
exceed 3200RPM?

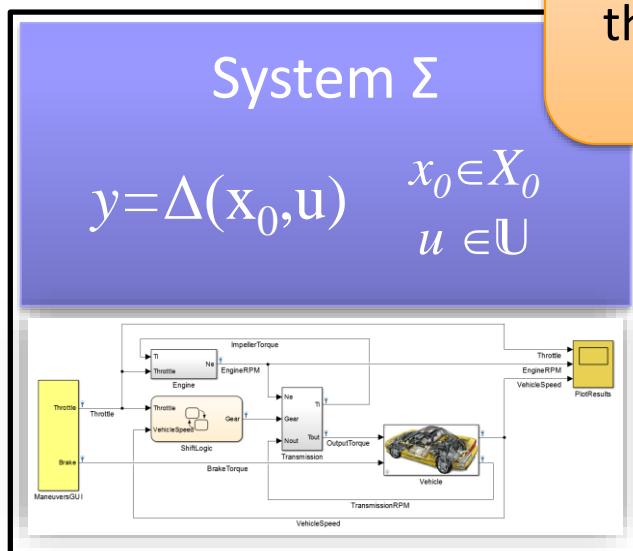


# Parameter Mining

What is the shortest time that the engine speed can exceed 3200RPM?



The vehicle speed is always less than parameter  $\theta_1$  and the engine speed is always less than  $\theta_2$ .

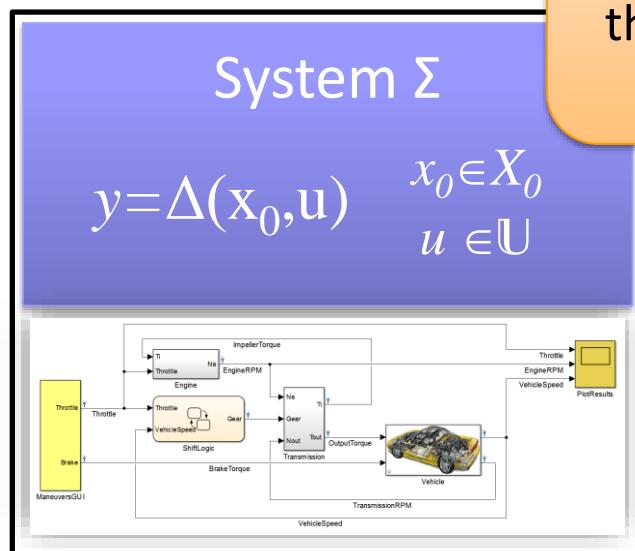


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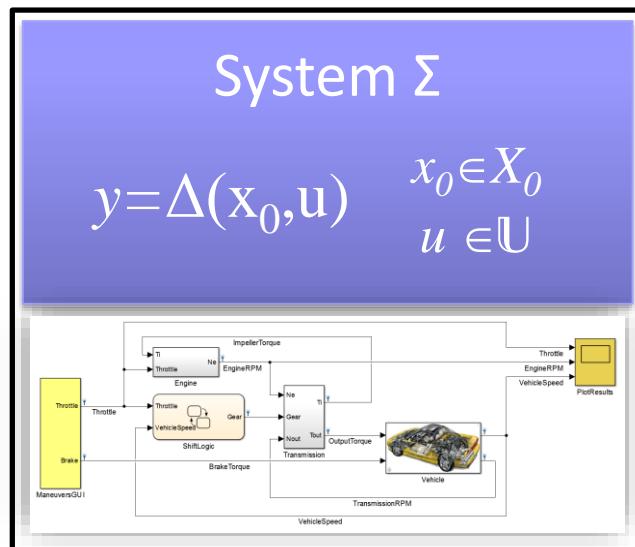


If I increase/decrease  $\theta_1$  by a specific amount, how much do I have to increase/decrease  $\theta_2$  so that the system satisfies the specification?"

# Parameter Mining

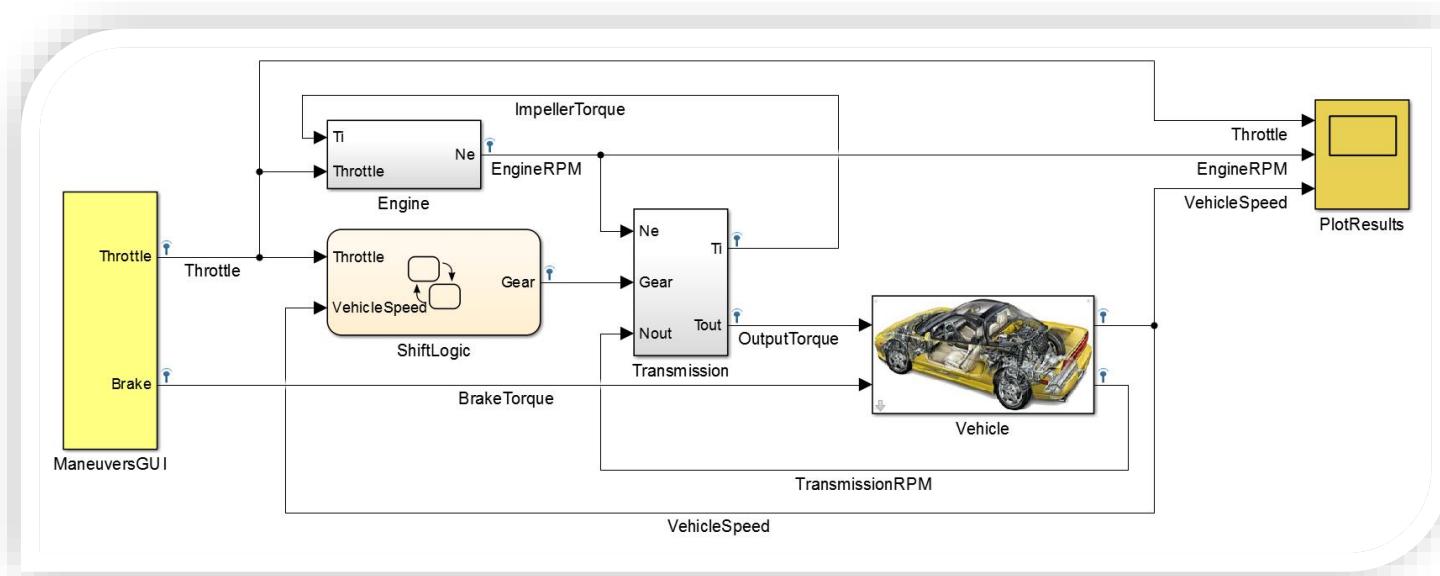
## Benefits:

- Facilitate the development of system specifications
  - In many cases, system requirements are not well formalized by the initial system design stages
- Explore and determine system properties
  - If a specification can be falsified, then it is natural to inquire for the range of parameter values that cause falsification.



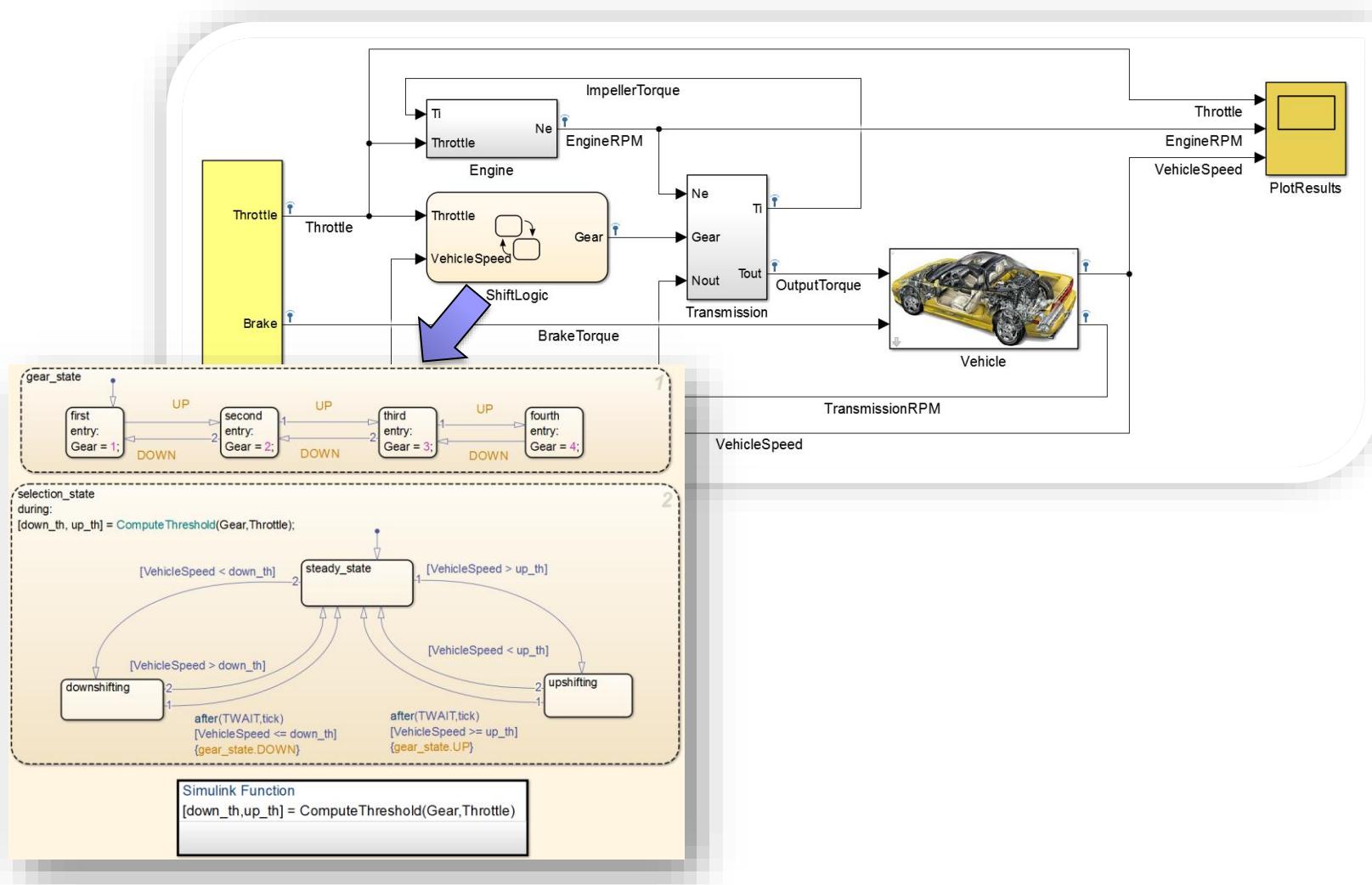
# Preliminaries – Running Example

## Automotive Transmission Simulink Demo



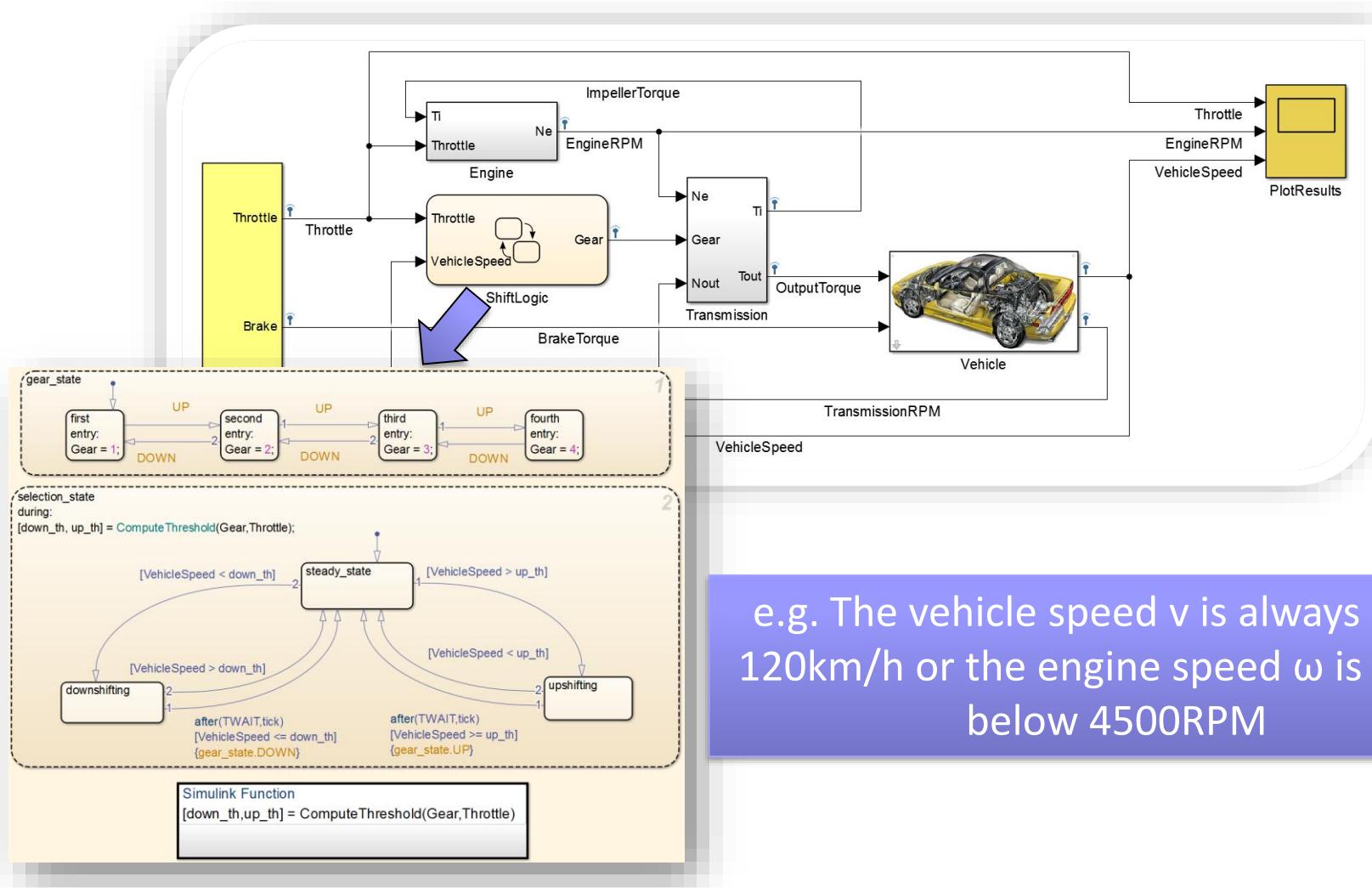
# Preliminaries – Running Example

## Automotive Transmission Simulink Demo



# Preliminaries – Running Example

## Automotive Transmission Simulink Demo

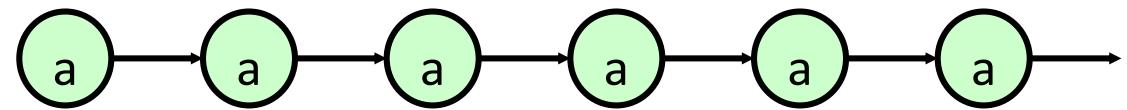


# Preliminaries - Metric Temporal Logic

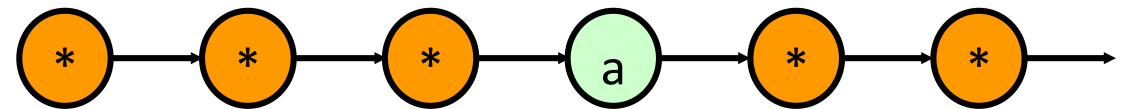
Syntax: Boolean connectives with temporal operators

$$\phi ::= \top \mid \neg\phi \mid \phi_1 \vee \phi_2 \mid G \phi \mid F \phi \mid \phi_1 U_I \phi_2$$

$G a$  - always a

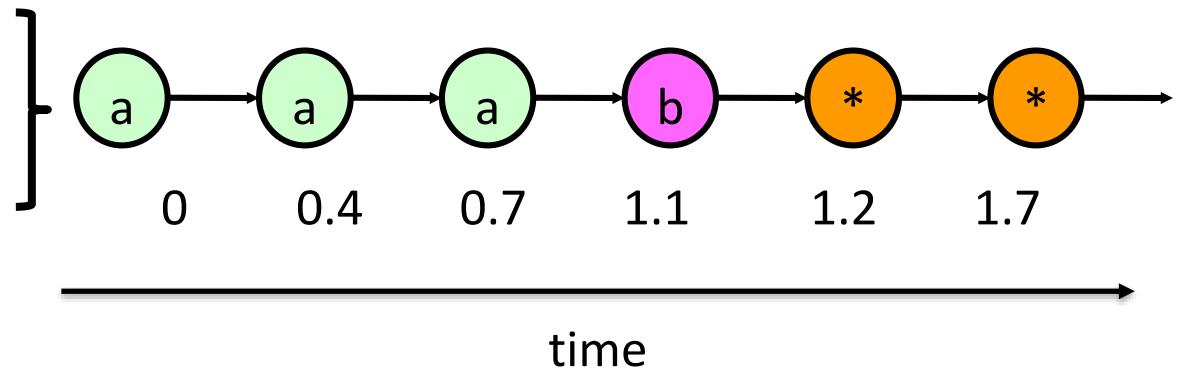


$F a$  - eventually a



$a U b$  - a until b

$a U_{[1,1.5]} b$  - a until b



Other notation:  $Ga \equiv \Box a$  and  $Fa \equiv \Diamond a$

# Parameter Mining

The vehicle speed is always less than parameter  $\theta_1$  and the engine speed is always less than  $\theta_2$ .



$$\text{Parametric MTL: } \phi_1[\vec{\theta}] = \square((v \leq \theta_1) \wedge (\omega \leq \theta_2))$$

PMTL formulas may contain state and/or timing parameters

$$\text{Ex. } \phi_2[\vec{\theta}] = \neg(\diamond_{[0, \theta_1]}(v > 100) \wedge (\omega \leq \theta_2))$$

Timing

State

# Parameter Mining

Parameter Mining Problem:

Given a parametric MTL formula  $\phi[\vec{\theta}]$  with a vector of  $m$  unknown parameters  
and a system  $\Sigma$ , find the set  $\Psi = \{\theta^* \in \Theta \mid \Sigma \not\models \phi[\theta^*]\}$

# Parameter Mining

Parameter Mining Problem:

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Question:

Why don't we search for the set of parameters for which the system satisfies the specification?

# Parameter Mining

Parameter Mining Problem:

Given a parametric MTL formula  $\phi[\vec{\theta}]$  with a vector of  $m$  unknown parameters and a system  $\Sigma$ , find the set  $\Psi = \{\theta^* \in \Theta \mid \Sigma \models \phi[\theta^*]\}$

Approximation possible ☺

Question:

Why don't we search for the set of parameters for which the system satisfies the specification?

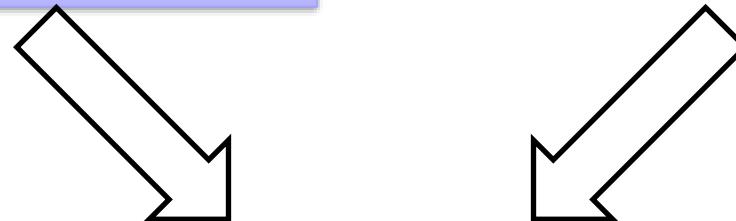
Problem is undecidable [AL94] ☹.

[AL94]: Alur, Rajeev, et al. "The algorithmic analysis of hybrid systems." *11th International Conference on Analysis and Optimization of Systems Discrete Event Systems*. Springer Berlin Heidelberg, 1994.

# Parameter Mining

*Testing framework based  
on the theory of robustness of  
MTL formulas*

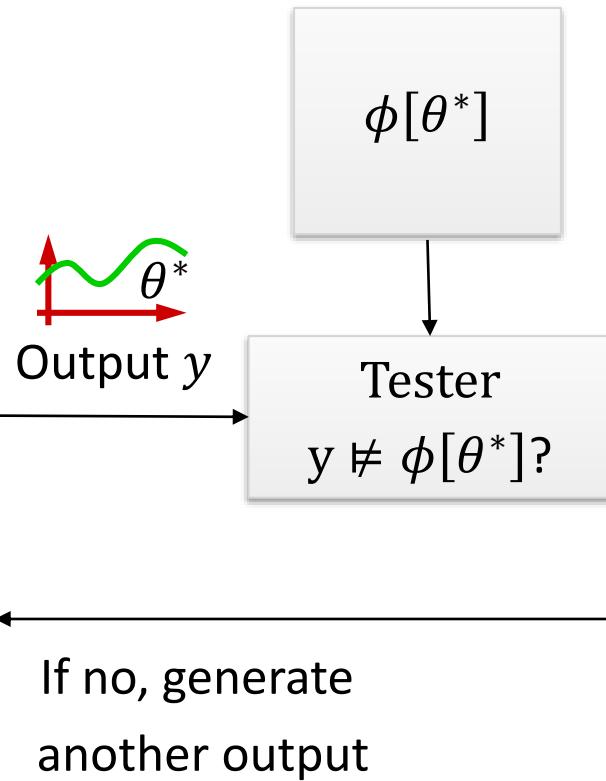
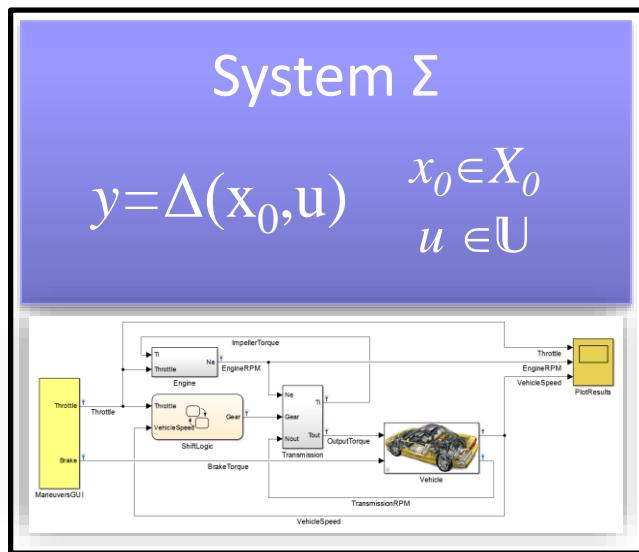
*Monotonicity properties of  
parametric MTL formulas.*



*Parameter mining ->  
Optimization problem*

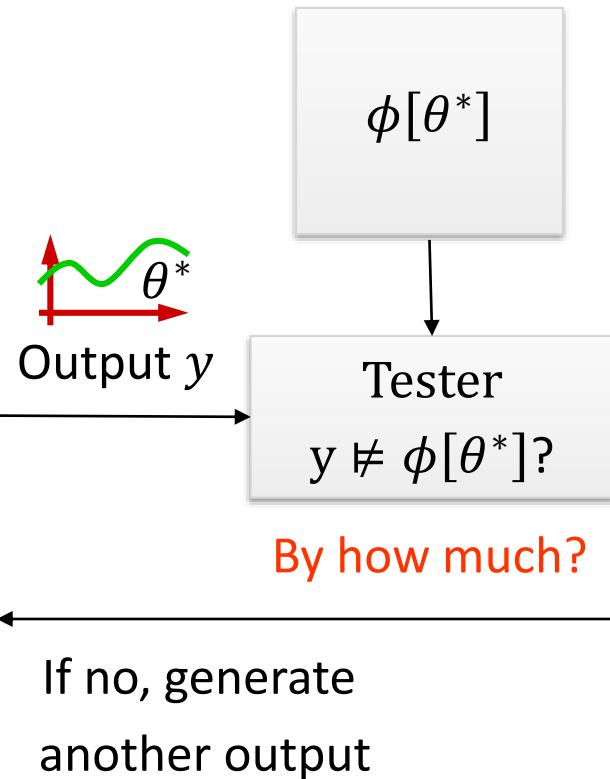
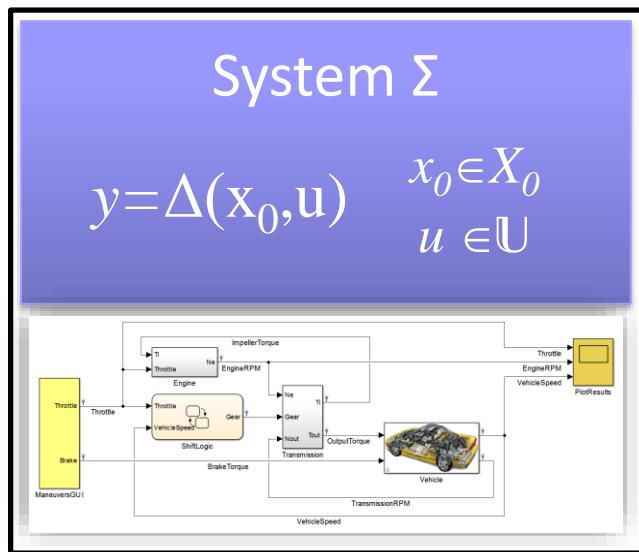
# Output Trajectory Testing

For a specific parameter valuation  $\theta^*$ :

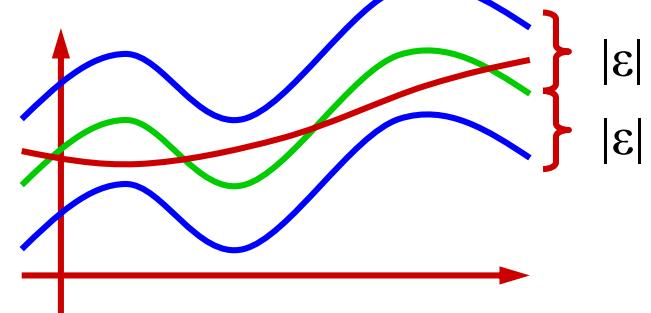
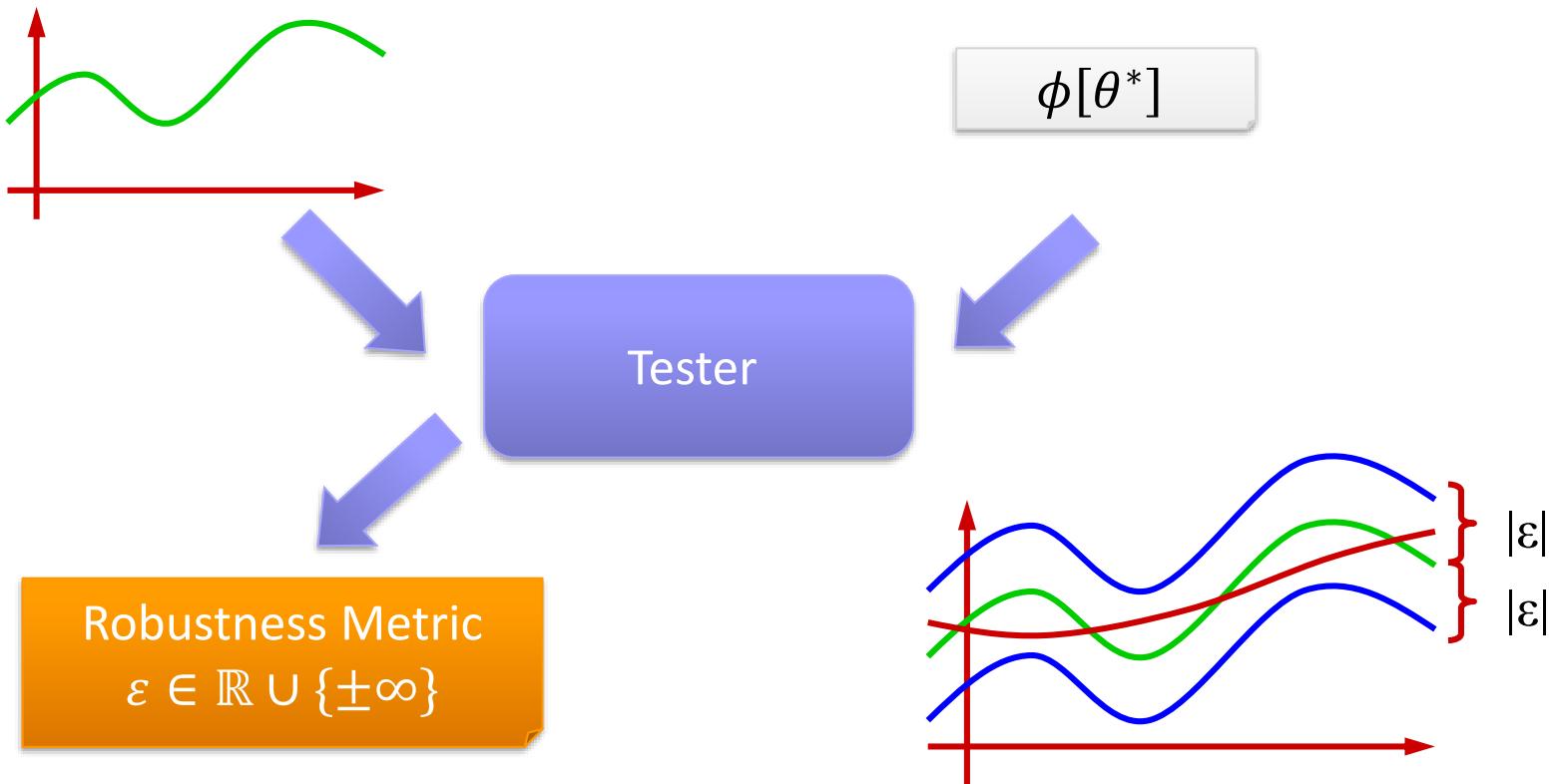


# Output Trajectory Testing

For a specific parameter valuation  $\theta^*$ :



# Robustness of Temporal Logics



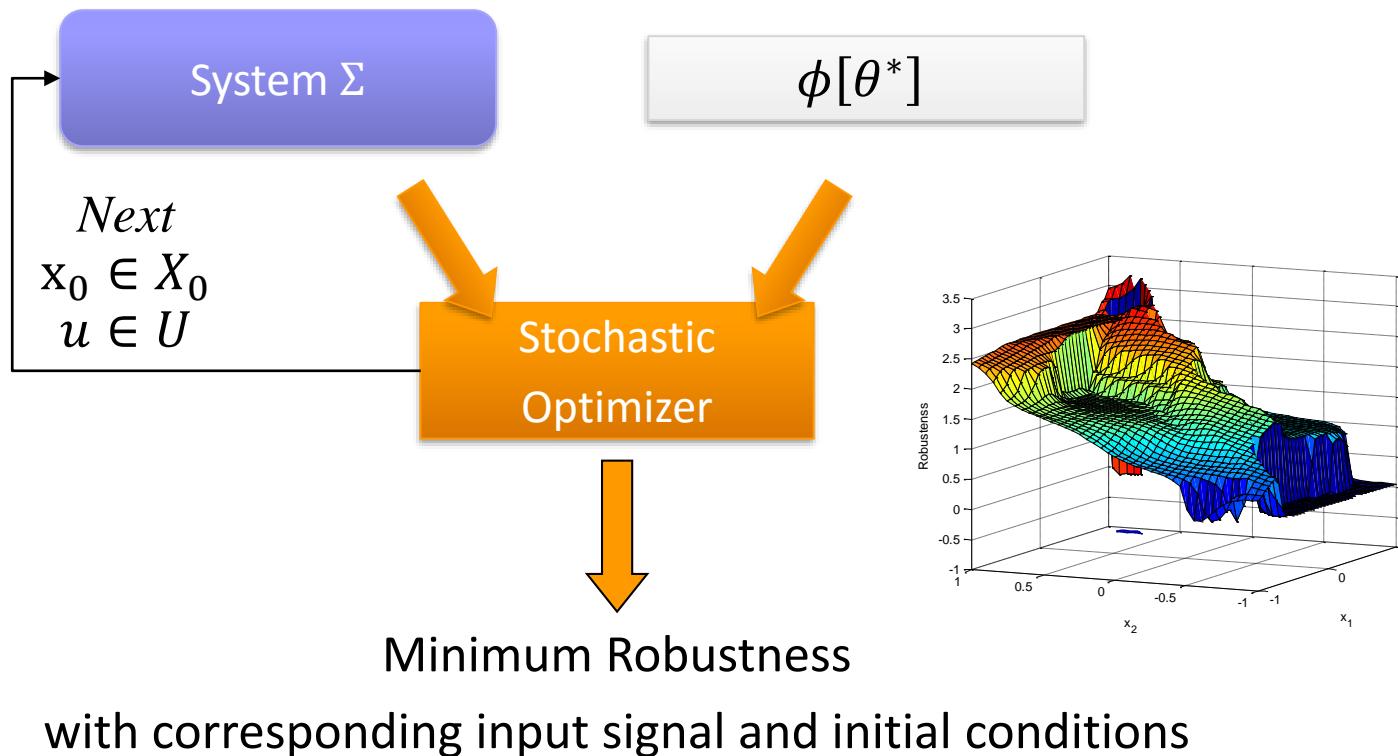
positive robustness  $\rightarrow$  signal satisfies the formula

negative robustness  $\rightarrow$  signal falsifies the formula

Fainekos and Pappas, *Robustness of temporal logic specifications for continuous-time signals*, Theoretical Computer Science, 2009

# Falsification by optimization

The falsification method searches for counterexamples that prove that the system does not satisfy the specification

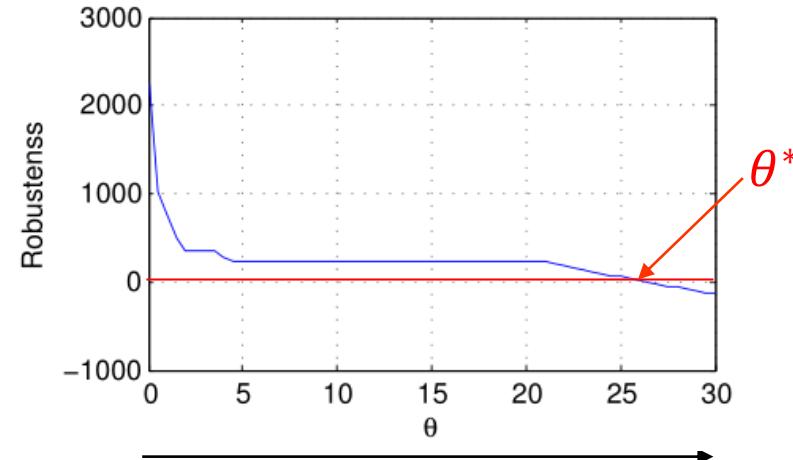
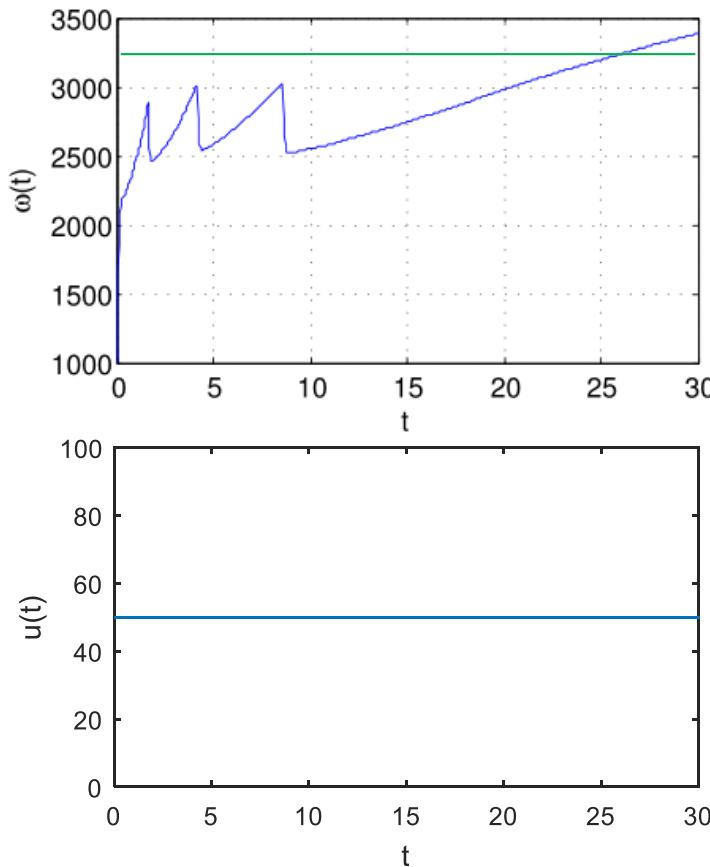


Abbas, et al, Probabilistic Temporal Logic Falsification of Cyber-Physical Systems, ACM TECS 2013

# Monotonicity of parametric MTL specifications

NL: Always, from 0 to  $\theta$ , the engine speed is less than 3250

$$\phi[\theta] = \square_{[0,\theta]}(\omega \leq 3250)$$

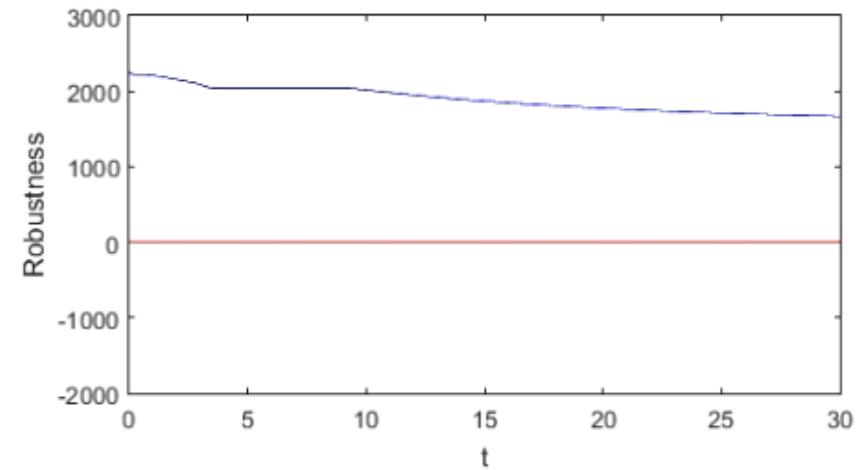
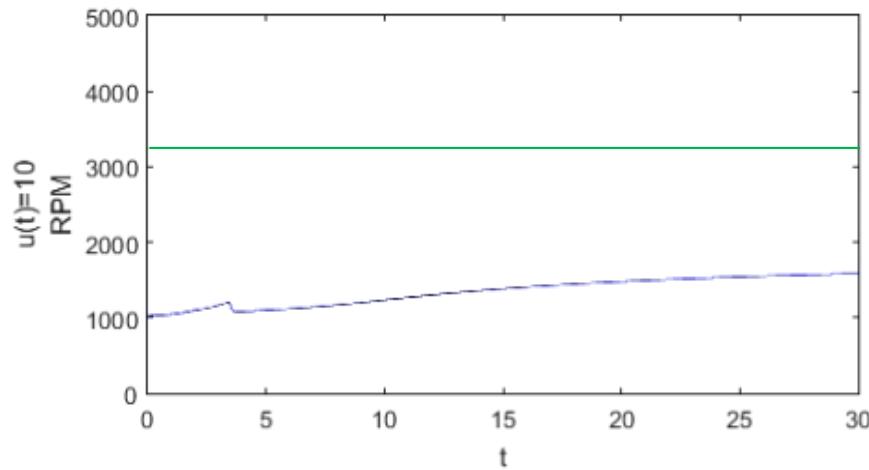


As we increase  $\theta$ , we can only increase the opportunity to find falsifying system behavior

Non-Increasing robustness with respect to  $\theta$

# Monotonicity of parametric MTL specifications

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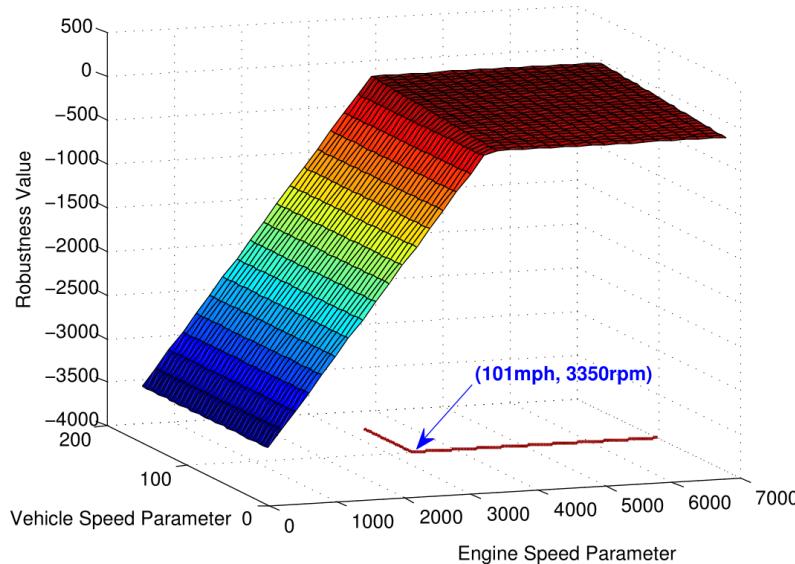


Monotonicity results formalized in  
[Hoxha, Dokhanchi, and Fainekos, arXiv:1512.07956]

# Monotonicity of parametric MTL specifications

NL: Always, vehicle speed is less than  $\theta_1$  and engine speed is less than  $\theta_2$

$$\phi_1[\theta] = \square((v \leq \theta_1) \wedge (\omega \leq \theta_2))$$



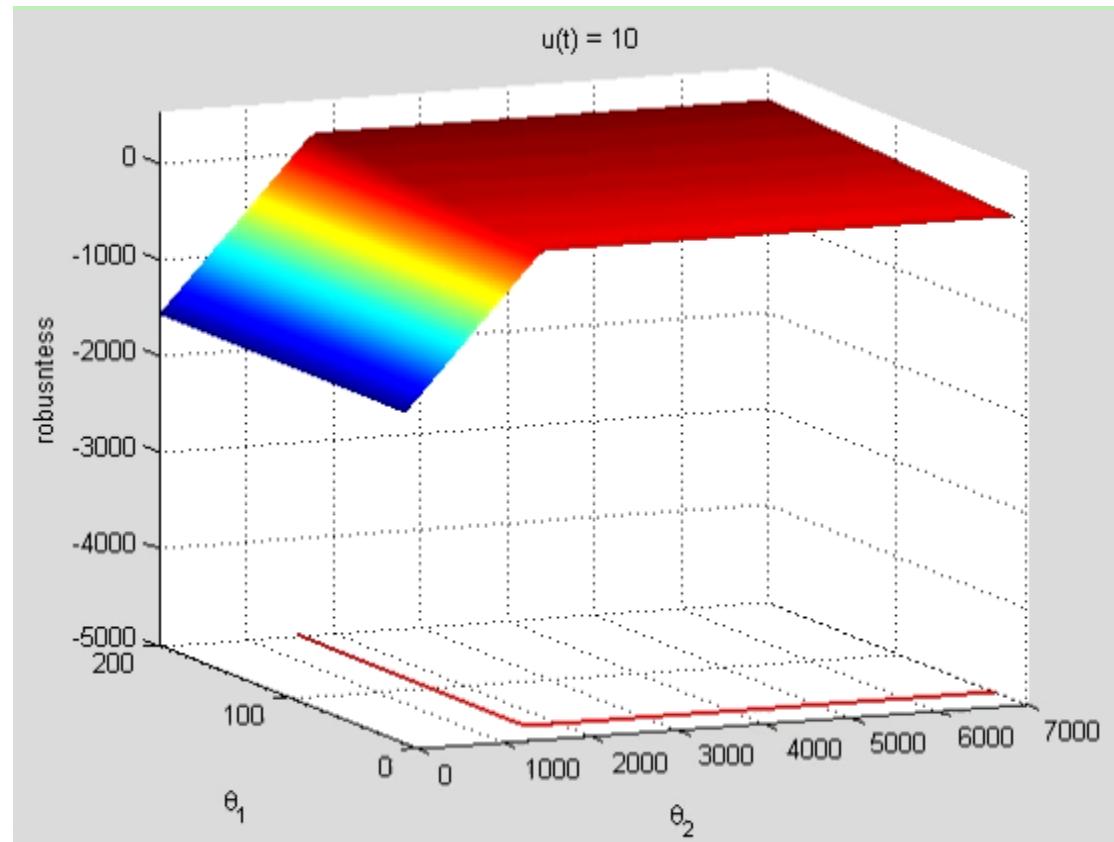
As we increase  $\theta_1$  and  $\theta_2$ , we can only decrease the opportunity to find falsifying system behavior

Non-Decreasing robustness with respect to  $f(\vec{\theta})$

Monotonicity results formalized in  
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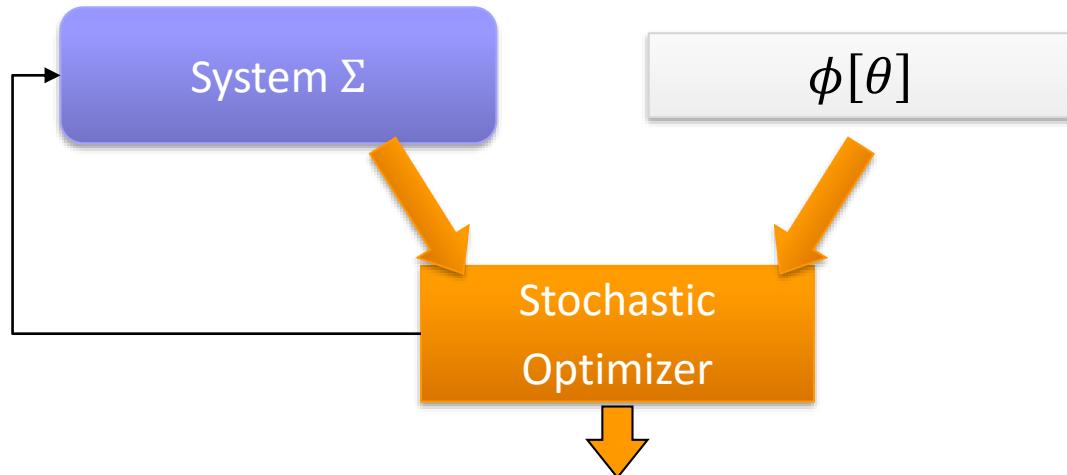
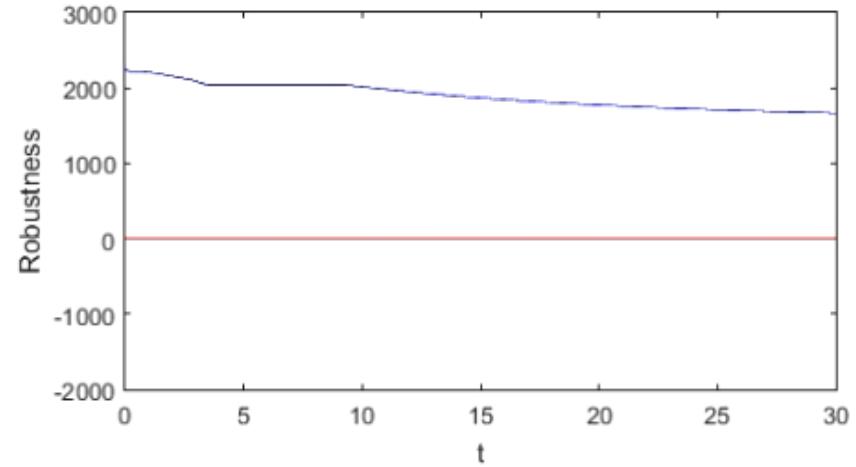
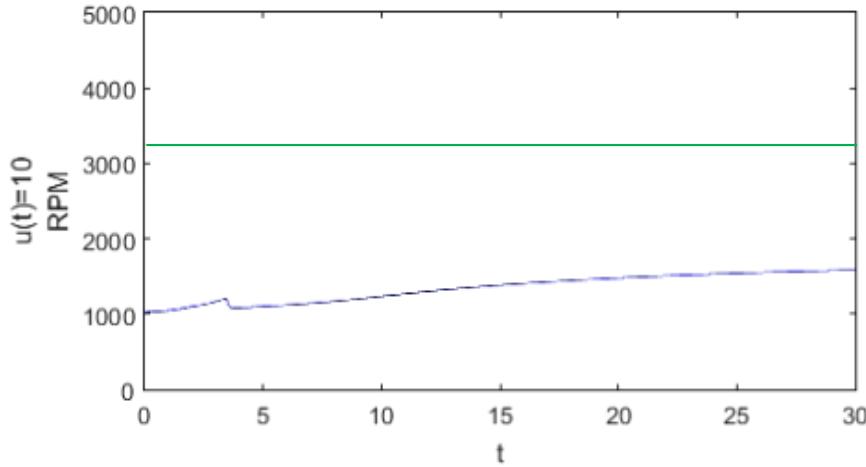
# Monotonicity of parametric MTL specifications

$$\phi_1[\theta] = \square((\nu \leq \theta_1) \wedge (\omega \leq \theta_2))$$



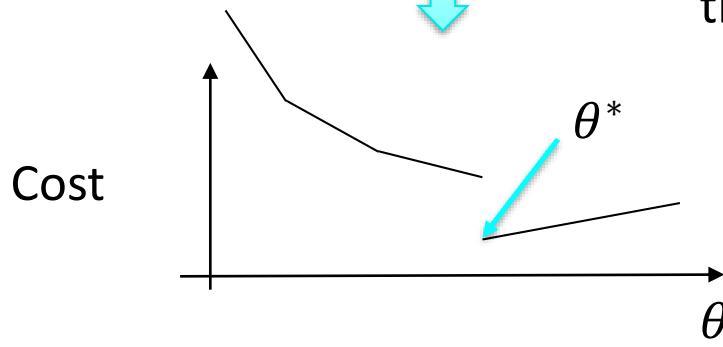
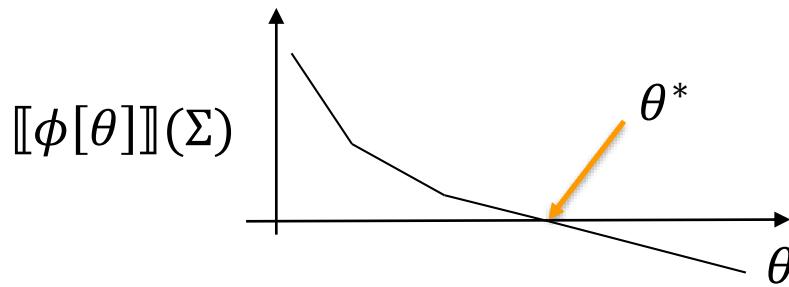
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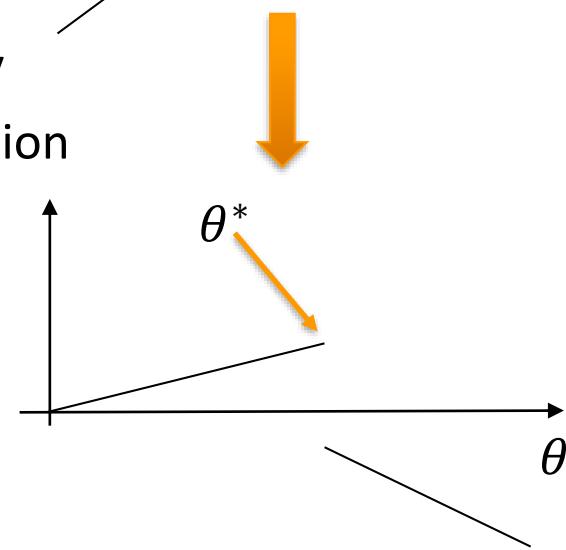
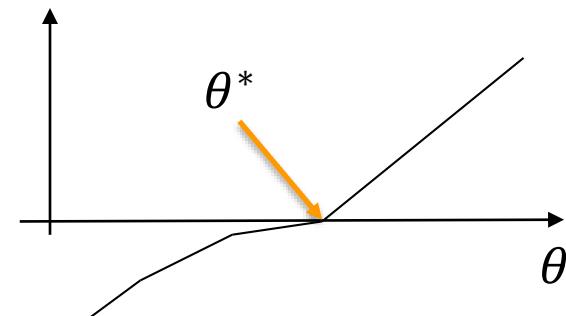


Solution to the Parameter Mining Problem.  
 Namely, set  $\Psi = \{\theta^* \in \Theta \mid \Sigma \not\models \phi[\theta^*]\}$

## Parameter Bound Computation



We modify  
the cost function



Non-Increasing robustness with respect to  $\theta$

**Minimize**

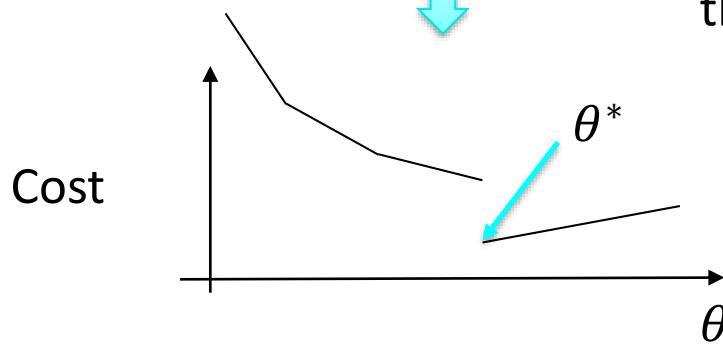
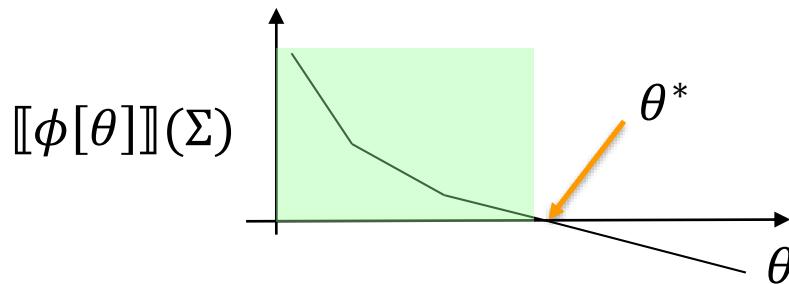
$$\min_{\theta \in \Theta} \min_{\mu \in \mathcal{L}_r(\Sigma)} \left( f(\theta) + \begin{cases} \gamma + [\phi[\theta]](\mu) & \text{if } [\phi[\theta]](\mu) \geq 0 \\ 0 & \text{otherwise} \end{cases} \right)$$

Non-Decreasing robustness with respect to  $\theta$

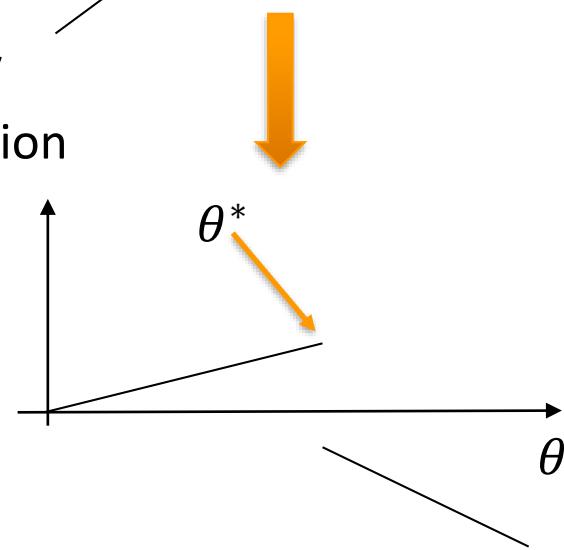
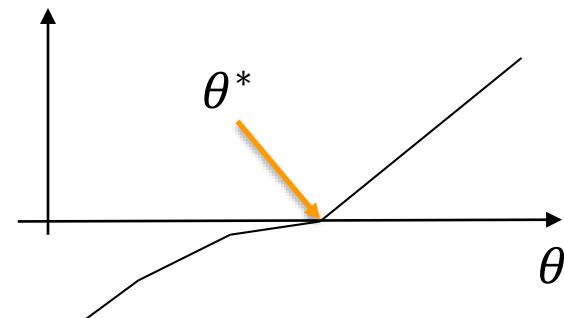
**Maximize**

$$\max_{\theta \in \Theta} \max_{\mu \in \mathcal{L}_r(\Sigma)} \left( f(\theta) + \begin{cases} \gamma - [\phi[\theta]](\mu) & \text{if } [\phi[\theta]](\mu) \geq 0 \\ 0 & \text{otherwise} \end{cases} \right)$$

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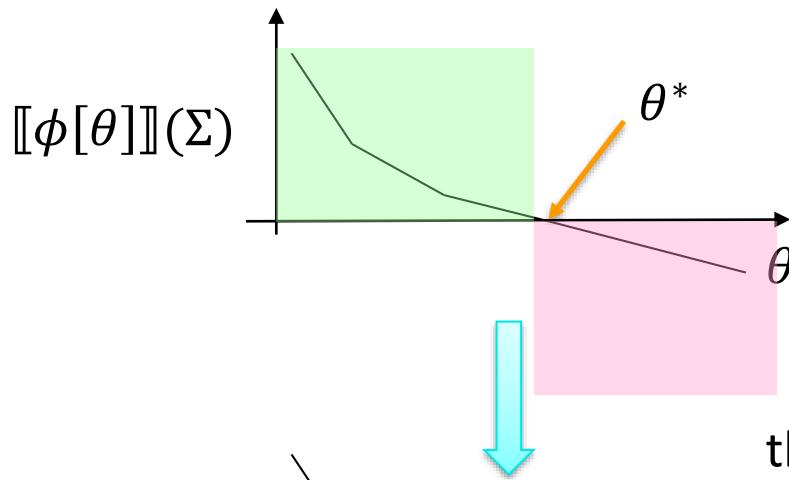
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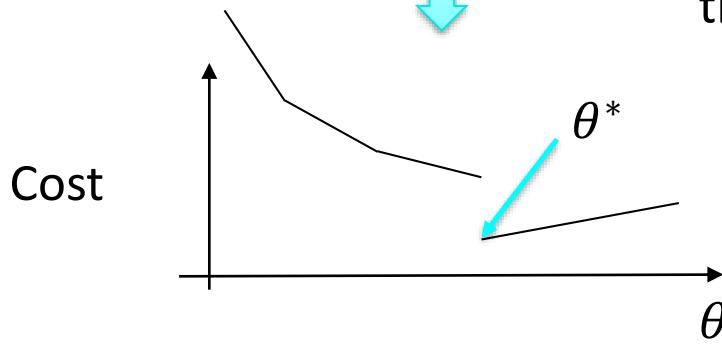
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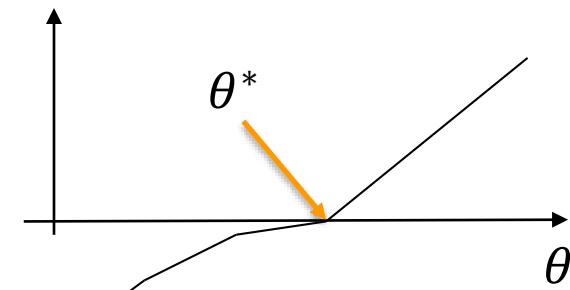
We modify  
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Non-Increasing robustness with respect to  $\theta$

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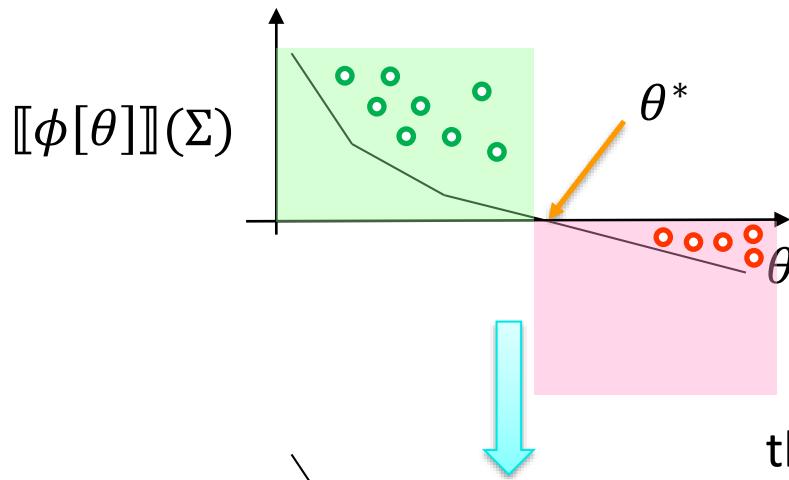


Non-Decreasing robustness with respect to  $\theta$

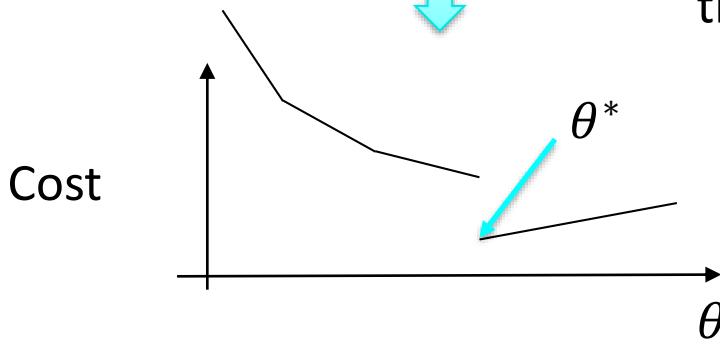
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## Parameter Bound Computation



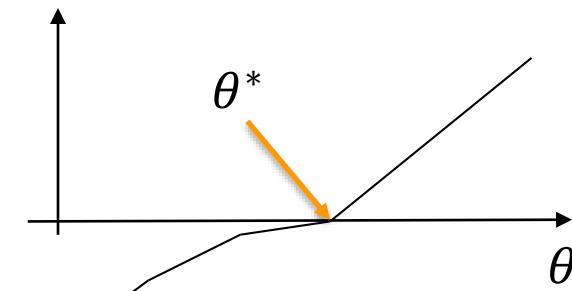
We modify  
the cost function



Non-Increasing robustness with respect to  $\theta$

**Minimize**

$$\min_{\theta \in \Theta} \min_{\mu \in \mathcal{L}_r(\Sigma)} \left( f(\theta) + \begin{cases} \gamma + \|\phi[\theta]\|(\mu) & \text{if } \|\phi[\theta]\|(\mu) \geq 0 \\ 0 & \text{otherwise} \end{cases} \right)$$

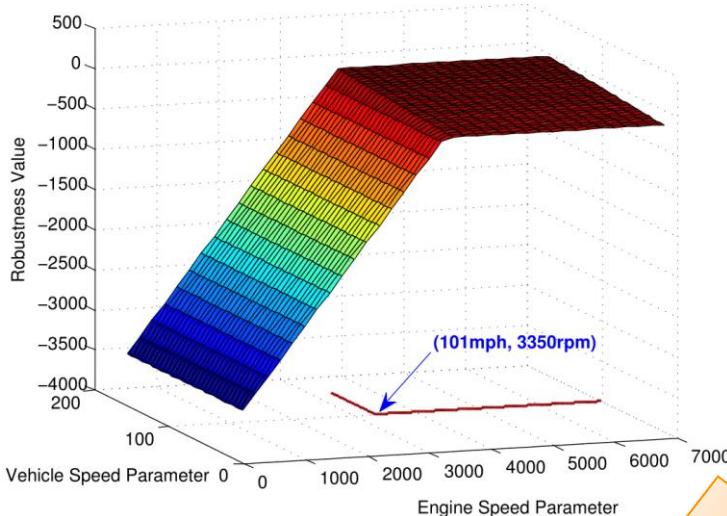


Non-Decreasing robustness with respect to  $\theta$

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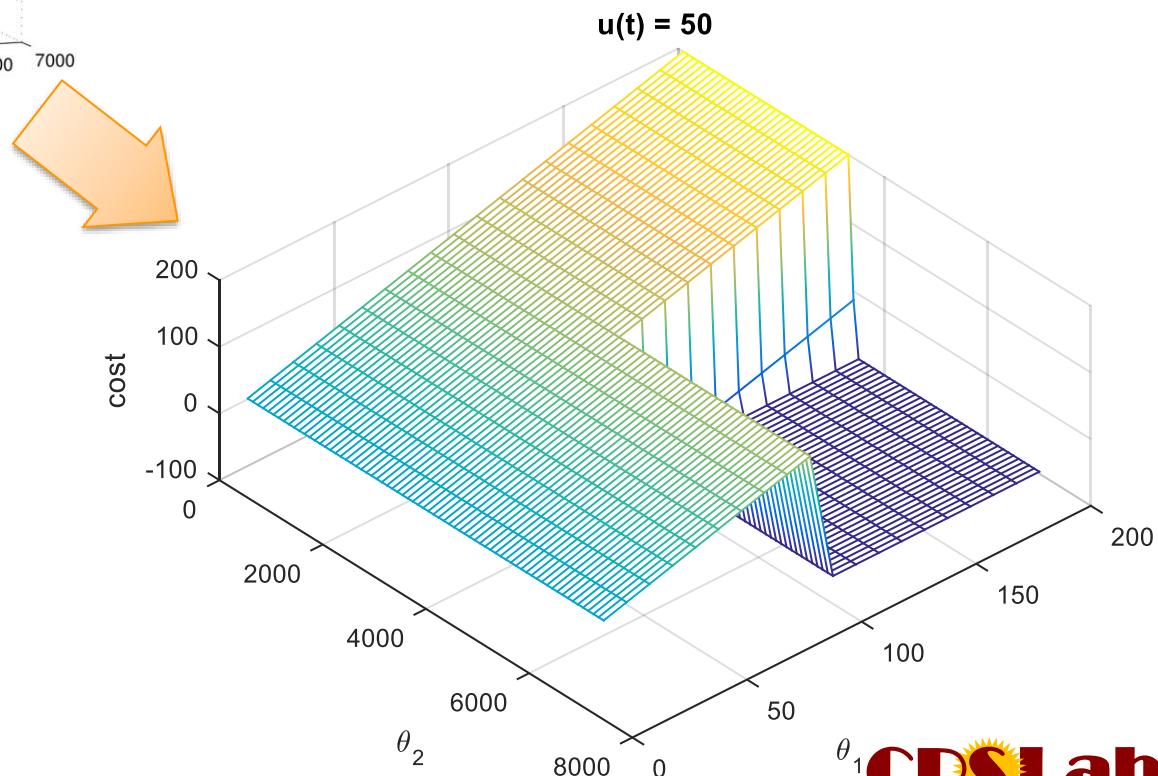
$$\max_{\theta \in \Theta} \max_{\mu \in \mathcal{L}_r(\Sigma)} \left( f(\theta) + \begin{cases} \gamma - \|\phi[\theta]\|(\mu) & \text{if } \|\phi[\theta]\|(\mu) \geq 0 \\ 0 & \text{otherwise} \end{cases} \right)$$

# Parameter Bound Computation



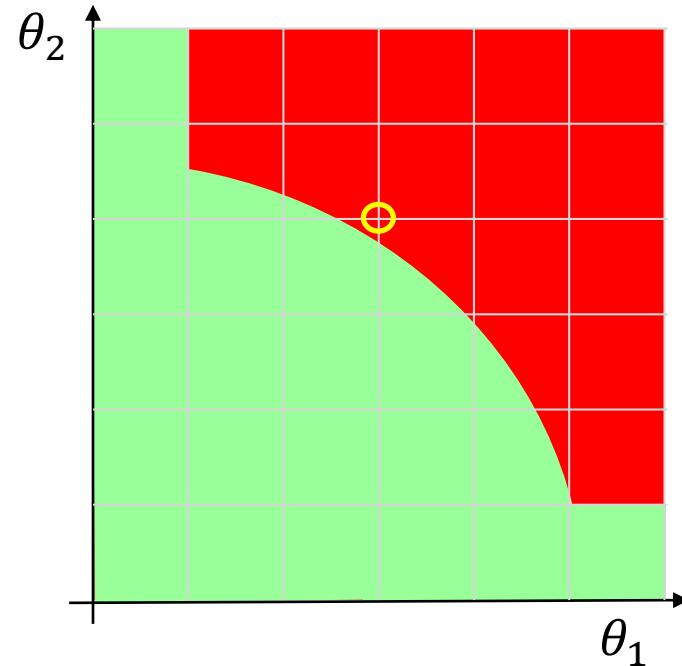
Non-Decreasing robustness with respect to  $f(\vec{\theta})$

$$\max_{\theta \in \Theta} \max_{\mu \in \mathcal{L}_\tau(\Sigma)} \left( f(\theta) + \begin{cases} \gamma - [\phi[\theta]](\mu) & \text{if } [\phi[\theta]](\mu) \geq 0 \\ 0 & \text{otherwise} \end{cases} \right)$$



# Parameter Falsification Domain

Non-Increasing robustness with respect to  $\theta$



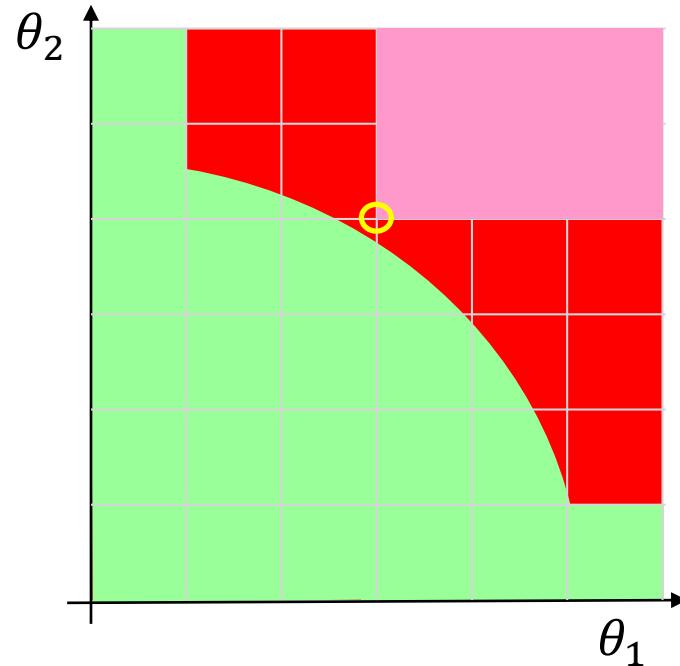
System fails the specification with  $\theta_1$  and  $\theta_2$



System satisfies the specification with  $\theta_1$  and  $\theta_2$

# Parameter Falsification Domain

Non-Increasing robustness with respect to  $\theta$



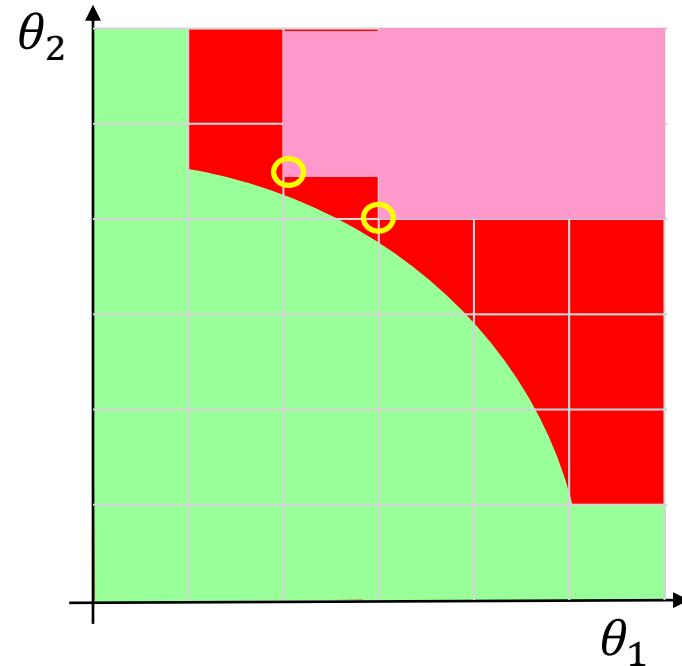
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# Parameter Falsification Domain

Non-Increasing robustness with respect to  $\theta$



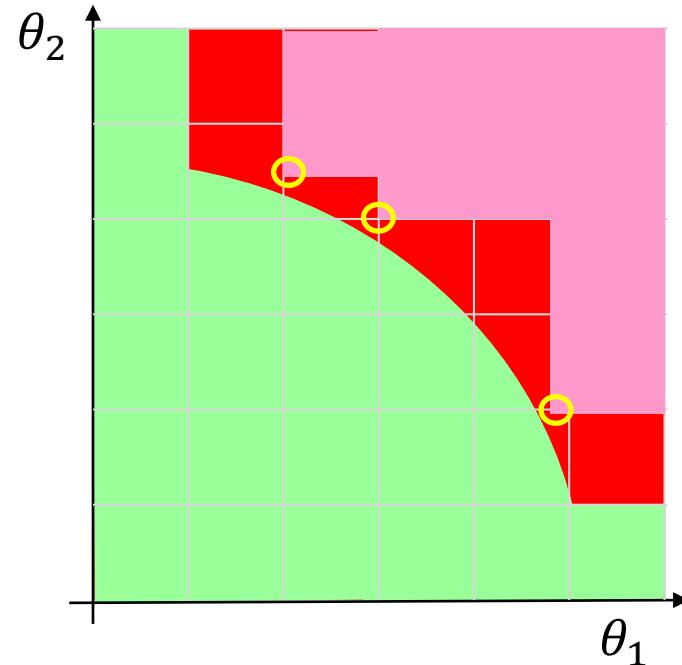
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# Parameter Falsification Domain

Non-Increasing robustness with respect to  $\theta$



System fails the specification with  $\theta_1$  and  $\theta_2$



System satisfies the specification with  $\theta_1$  and  $\theta_2$

# Parameter Falsification Domain

Alg 1: Robustness Guided Parameter Falsification Domain Algorithm

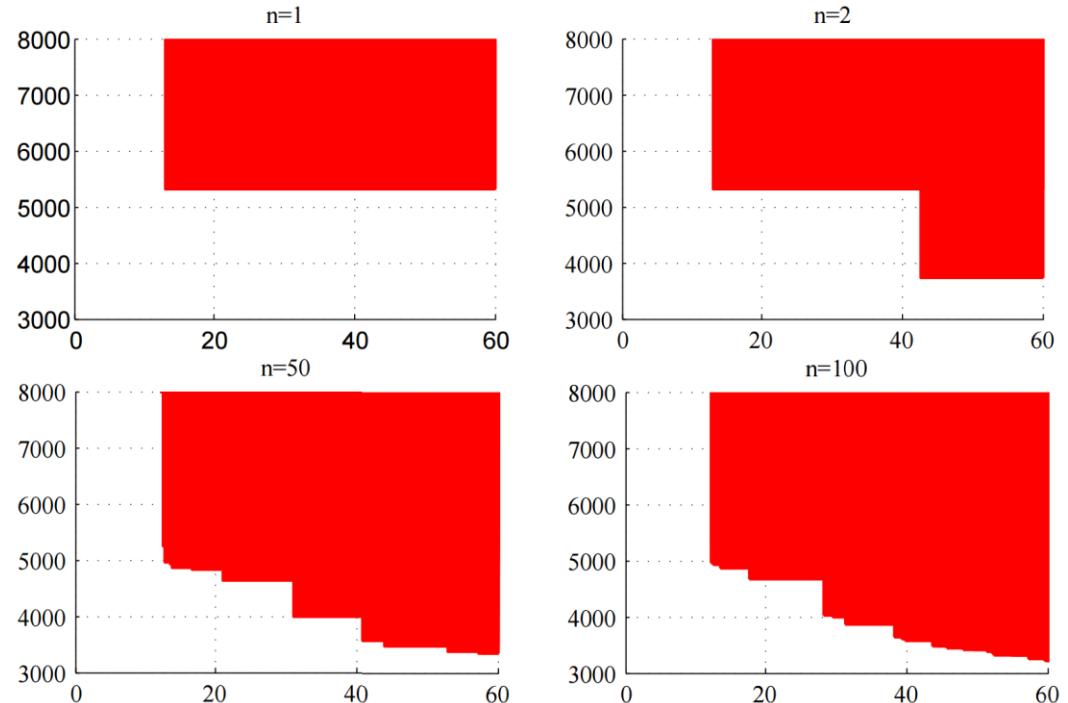
$$\phi[\theta] = \neg(\Diamond_{[0, \theta_1]}(\nu \geq 100) \wedge \Box(\omega \leq \theta_2))$$

Non-Increasing robustness with respect to  $f(\theta)$

In each iteration, shift weights of the priority function

$f(\theta) = \sum w_i \theta_i$ , which shifts the minimum of the cost function

$$\min_{\theta \in \Theta} \min_{\mu \in \mathcal{L}_\tau(\Sigma)} \left( f(\theta) + \begin{cases} \gamma + \llbracket \phi[\theta] \rrbracket(\mu) & \text{if } \llbracket \phi[\theta] \rrbracket(\mu) \geq 0 \\ 0 & \text{otherwise} \end{cases} \right)$$



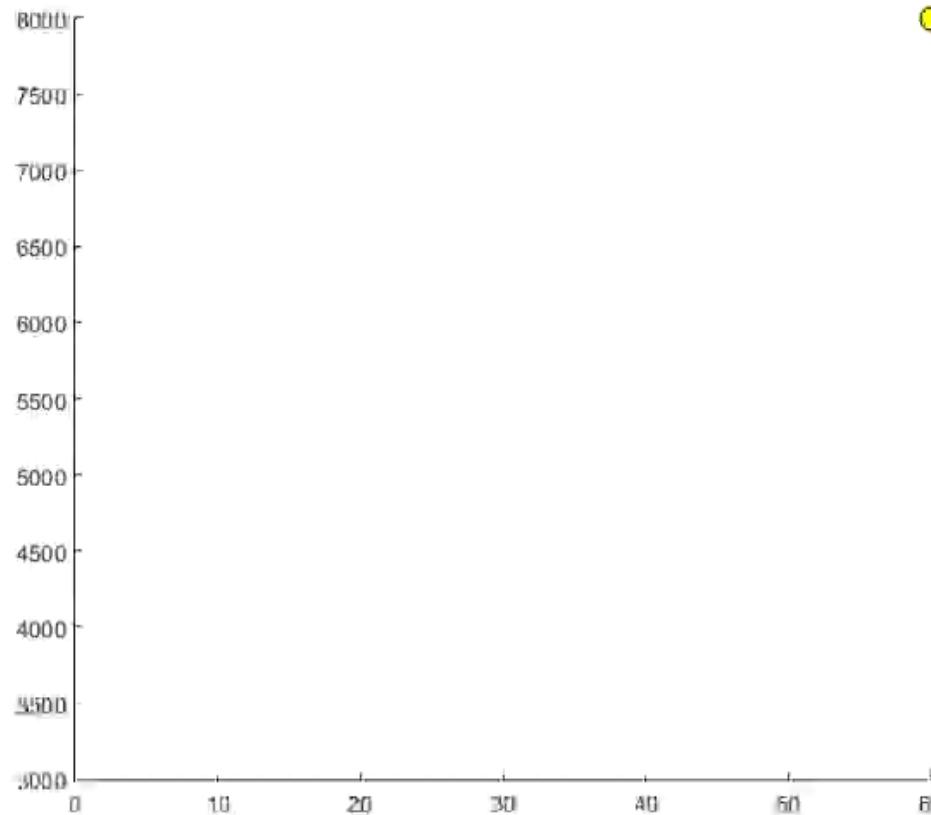
Red Colored Set represents the parameter falsification domain

# Parameter Falsification Domain

Alg 1: Robustness Guided Parameter Falsification Domain Algorithm

$$\phi[\theta] = \neg(\Diamond_{[0, \theta_1]}(v \geq 100) \wedge \Box(\omega \leq \theta_2))$$

Non-Increasing robustness with respect to  $f(\theta)$

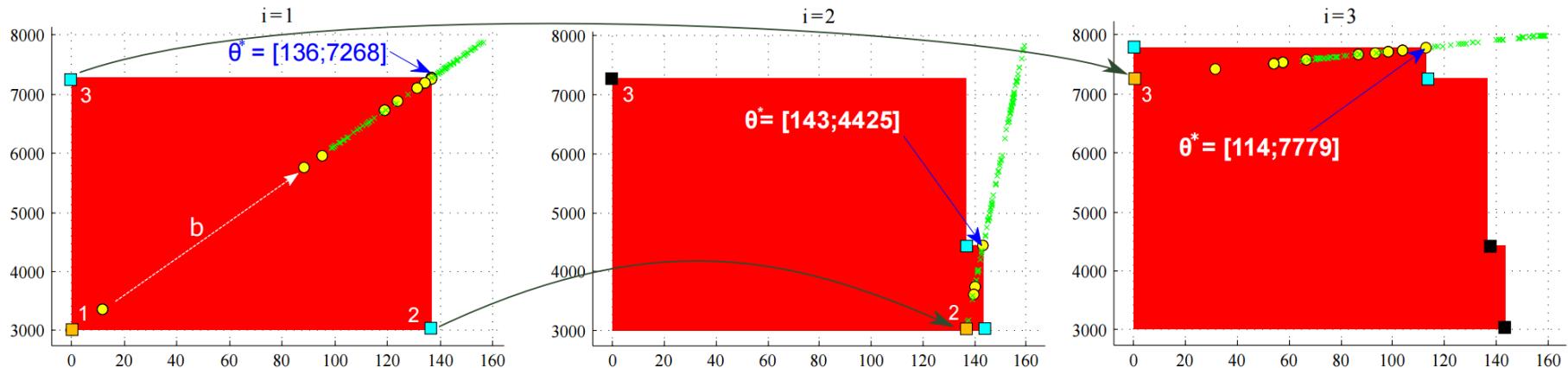


# Parameter Falsification Domain

Alg 2: Structured Parameter Falsification Domain Algorithm

$$\phi[\theta] = \square((v \leq \theta_1) \wedge (\omega \leq \theta_2))$$

Non-Decreasing robustness with respect to  $f(\vec{\theta})$



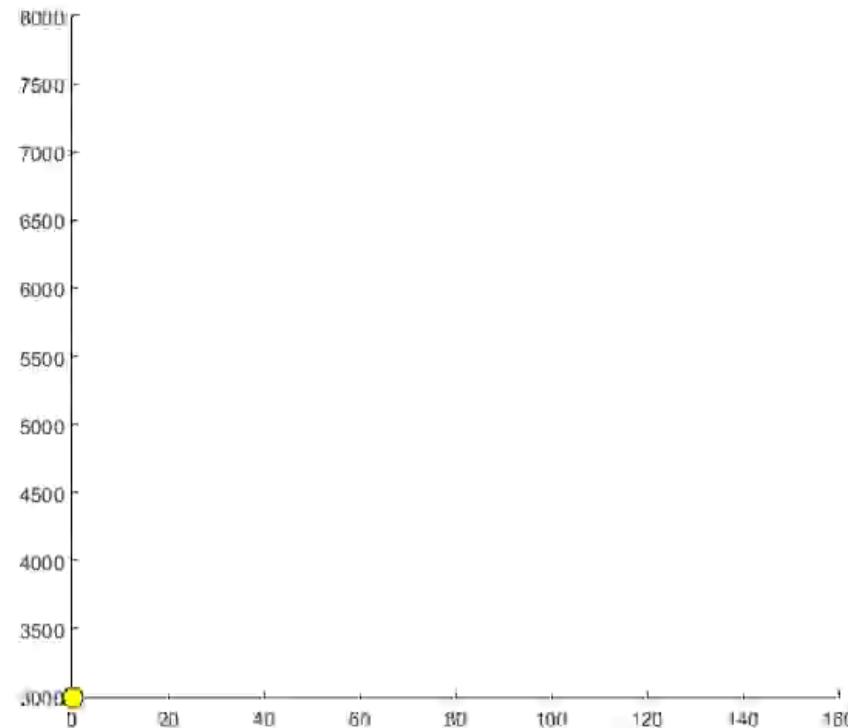
$$\max_{\theta \in \Theta} \max_{\mu \in \mathcal{L}_\tau(\Sigma)} \left( f(\theta) + \begin{cases} \gamma - [\phi[\theta]](\mu) & \text{if } [\phi[\theta]](\mu) \geq 0 \\ 0 & \text{otherwise} \end{cases} \right)$$

# Parameter Falsification Domain

Alg 2: Structured Parameter Falsification Domain Algorithm

$$\phi[\theta] = \square((v \leq \theta_1) \wedge (\omega \leq \theta_2))$$

Non-Decreasing robustness with respect to  $f(\vec{\theta})$



# Related Works

Parametric temporal logics over:

- Finite State Machines:
  - Alur et al. Parametric temporal logic for model measuring, 2001
- Timed Automata:
  - R. Alur et al. Parametric real-time reasoning, 1993
  - Bozzelli and La Torre. Decision problems for lower/upper bound parametric timed automata, 2009
- Hybrid Systems:
  - Asarin et al. Parametric identification of temporal properties, 2012.
  - Jin et al. Mining requirements from closed loop control models, 2013.

# Conclusions

- We extend and generalize the parameter mining problem presented in [Yang, Hoxha and Fainekos, Querying Parametric Temporal Logic Properties on Embedded Systems, 2012].
- We present two algorithms to explore the Pareto front of parametric MTL with multiple parameters.
- The algorithms presented in this work are publicly available through our toolbox S-TaLiRo.

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# Thank you!

# Questions?